Unit 1: Logical Operations



| Term | Definition | |
|---------------------|--|--|
| Propositional logic | A proposition is a simply a statement. | |
| | Propositional statements when evaluated will result in either true or false. | |
| | Propositional logic considers the way statements interact with each other. | |
| | Propositional logic follows mathematical rules. | |
| | | |

Logic statements

Most rules to simplify a logic statement are not dependent on the contents of the statement but on the structure of the statement.

Propositional logic uses symbols to represent logical links between propositions. A logic statement includes propositions linked connected by logical links.

| Term | Definition | |
|-----------------------------|---|---------------|
| Propositional logic symbols | Propositional logic uses symbols to represent logical links | |
| Symbol | Formal term | Informal term |
| • | Connection | AND |
| + | Separation | OR |
| Ā | Negation | NOT |
| \oplus | Exclusive separation | XOR |

| Term | Definition |
|-------------|--|
| Truth table | A truth table is a mathematical table used to analyse a set of local statements. |

Connection (AND)

"I am hungry and I have a sandwich"

For this logical statement to be true both propositions would have to be \mathtt{TRUE} . The key word is \mathtt{AND} .

If one proposition was false, then the whole statement would be false. Let's use A to represent "I am hungry" and B to represent "I have a sandwich".

A truth table for this can show all the possibilities using 1 for TRUE and 0 for FALSE.

| А | В | A AND B |
|---|---|---------|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

A AND B can be written using a symbol as A.B

Separation (OR)

"She has blue eyes or she has brown eyes"

Sometimes one proposition or another proposition of a logical statement is correct. The key word here is OR.

This time the logical statement will be true if at least one proposition is TRUE.

| Α | В | A OR B |
|---|---|--------|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

A OR B can be written using a symbol as A + B

Negation (NOT)

"I am 16 years old"

To negate this proposition, "I am not 16 years old" we can use the negation operator. A truth table can show this:

| Α | Ā |
|---|---|
| 1 | 0 |
| 0 | 1 |

NOT A can be written using a symbol as Ā.

Exclusive separation (XOR)

"She has blue eyes or she has brown eyes"

For exclusive separation only one proposition of a logical statement can be correct.

A truth table can show this:

| Α | В | A XOR B |
|---|---|---------|
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

A XOR B can be written using a symbol as $\mathsf{A} \oplus \mathsf{B}$

Zebras are black and white.
Pandas are black and white.
Therefore, some zebras are pandas.