Unit 1: Boolean algebra

Terminology

- TRUE will be represented by 1
- FALSE will be represented by 0
- Variables will be single letters e.g. A
- Logical operator OR will be +
  A+B represents A OR B
- Logical operator AND will be •
  A.B represents A AND B
- Logical operator NOT will be an overbar
  $\bar{A}$ represents NOT A
- Logical operator XOR will be ⊕
  A⊕B represents A XOR B

Operation	Definition
OR	Takes two inputs and outputs 1 (true) if either is true
AND	Takes two inputs and outputs 1 (true) if both are true
NOT	Takes one input and outputs the opposite value
XOR	Takes two inputs and outputs 1 if only one of the inputs is true

Order of precedence

There is an order of precedence for operations in Boolean algebra just like BIDMAS is used in mathematical algebra.

Order of precedence

The order of precedence is (highest first):

Brackets
NOT
XOR
AND
OR

Commutative Law

The commutative laws of Boolean algebra are:

A.B = B.A

A+B = B+A

A⊕B = B⊕A

Associative Law

The associative laws of Boolean algebra are:

A.(B.C) = (A.B).C

A +(B+C) = (A+B) + C

A ⊕ (B⊕C) = (A⊕B) ⊕ C

Simplifying Boolean expressions

Using the OR operation:

- Identity law – A + 0 = A
- Annulment law – A + 1 = 1
- Idempotent law – A + A = A
- Inverse law – A + $\bar{A}$ = 1

Using the AND operator:

- Identity law – A.1 = A
- Annulment law – A.0 = 0
- Idempotent law – A.A = A
- Complement law – A.$\bar{A}$ = 0

Absorptive law

A + (A.B) = A

A	B	A.B	A + (A.B)
1	1	1	1
1	0	0	1
0	1	0	0
0	0	0	0

The truth table shows that the final column is A

Similarly, it can be shown that A.(A+B) = A

Working with brackets

The distributive law is used to expand brackets.

A.(B + C) = A.B + A.C

Factorising expressions can be done if terms have a common factor

A.B + A.C = A.(B + C)

Example question and solution

Clearly showing each step, simplify the following Boolean expressions using Boolean algebra and identities:

a. P.(0 + P) [1]

b. Q.(Q + P) + P.(Q + P) [4]

a. Using distributive law
P.(0+P) → P.0 + P.P
Using annulment law
P.0 → 0
Using Idempotent law
P.P → P
Therefore
P.(0.P) = P

b. Expand Brackets
Q.Q + Q.P + P.Q + P.P
Using idempotent law
Q + Q.P + P.Q + P
Factorising the expression
Q(1 + P) + P(Q +1)
Using annulment law
Q + P
Therefore
Q.(Q + P) + P.(Q + P) = Q + P