

Key Stage 4

Higher Algebra Revision



Name:

Teacher:

- (a) Write down the first three terms of the sequence whose n th term is given by $n^2 - 6$. [2]

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1st term = 2nd term = 3rd term =

- (b) Write down an expression for the n th term of the following sequence. [2]

5, 11, 17, 23, ...

The n th term of a sequence is given by $n^2 + 7$.

Write down the first three terms of this sequence. [2]

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1st term = 2nd term = 3rd term =

- (a) Write an expression for the n th term of the following sequence. [2]

2 7 12 17

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n th term =

- (a) Factorise $x^2 - 7x + 12$, and hence solve $x^2 - 7x + 12 = 0$. [3]

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- (b) Expand and simplify $(5x - 2)^2$. [2]

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(a) Write down the n th term of the following sequence. [2]

8, 11, 14, 17,

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(b) Make t the subject of the formula $r = 3t - 8$. [2]

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(c) A rectangle has a length of $(x + 5)$ cm and a width of $(2x - 3)$ cm.
Its perimeter is 46 cm.

Calculate the value of x . [4]

(a) $x^3 \times x^6 =$ [1]

x^{36}

$x^{0.5}$

x^2

x^9

x^{18}

.....

(b) $(7x - 5y) - (3x + 2y) =$ [1]

$4x - 3y$

$4x - 7y$

$4x + 3y$

$-4x + 7y$

$-4x - 7y$

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(c) A car travels x miles in 30 minutes.
Its average speed in miles per hour is [1]

$\frac{x}{2}$

$\frac{x}{30}$

$2x$

$\frac{2}{x}$

$30x$

(a) Factorise $x^3 - 5x$. [1]

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(b) Expand and simplify $(2x - 3)(x + 4)$. [2]

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(c) Factorise $x^2 - 3x - 28$. [2]

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Make c the subject of the following formula.
Give your answer in its simplest form. [5]

$$c - 5 = \frac{3c - 7}{d}$$

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Factorise $x^2 - 2x - 24$, and hence solve $x^2 - 2x - 24 = 0$. [3]

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Make x the subject of the following formula. [4]

$$a(x - b) = x(c - d)$$

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- (a) Circle the expression which is equivalent to $m^{\frac{4}{3}}$. [1]

$\frac{1}{3}m^2$

$2m^{\frac{1}{3}}$

$\frac{2}{3}m$

$(\sqrt[3]{m})^2$

$(\sqrt{m})^3$

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- (b) Circle the expression which is equivalent to $p^{\frac{3}{4}} \times p^{-\frac{1}{4}} \div p^{\frac{1}{4}}$. [1]

$p^{-\frac{1}{4}}$

$p^{-\frac{3}{64}}$

$p^{\frac{5}{4}}$

$p^{\frac{3}{4}}$

$p^{\frac{1}{4}}$

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The first four terms of a sequence are

3, 9, 19, 33,

Find the 100th term of the sequence. [3]

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- (a) Factorise $(x - 7)^2 + 2(x - 7)$. [2]

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- (b) Factorise $12x^2 - 27y^2$. [3]

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(a) Factorise $4x^2 - 81$.

[2]

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(b) Factorise $7x^2 + 10x - 8$.

[2]

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(c) Factorise $(x + 2)^3 + 5(x + 2)^2$.

[2]

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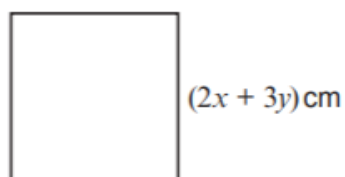
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Solve the following simultaneous equations using an algebraic (not graphical) method. [4]

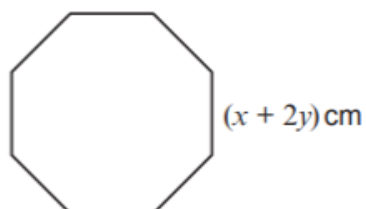
$$\begin{aligned}3x + 4y &= 7 \\ 2x - 3y &= 16\end{aligned}$$

In this question you will be assessed on the quality of your organisation, communication and accuracy in writing.

Each side of a square is of length $(2x + 3y)$ cm.
The perimeter of the square is 62 cm.



Each side of a regular octagon is of length $(x + 2y)$ cm.
The perimeter of the octagon is 72 cm.



Use an algebraic method to find the value of x and the value of y .

[5 + 2 OCW]

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Solve the following simultaneous equations using an algebraic (not graphical) method.

$$\begin{array}{r} 5x + 3y = 11 \\ 2x - 7y = 29 \end{array}$$

You must show all your working.

[4]

The Morgan family and the Smith family are on holiday in Aberystwyth.
There are 7 adults and 2 children in the Morgan family.
There are 4 adults and 3 children in the Smith family.

Both families visit a Craft Centre.
The entry price to the Craft Centre is £ x for adults and £ y for children.

The total cost for the Morgan family is £41.50.
The total cost for the Smith family is £29.75.

Form two equations in terms of x and y .

Solve your equations, using an algebraic method, to find the entry price for adults and the entry price for children. [5]

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Solve the following simultaneous equations using an algebraic (not graphical) method.

$$\begin{array}{r} 3x - 2y = 14 \\ 7x + 3y = 25 \end{array}$$

You **must** show all your working.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(a) Make m the subject of the formula $y = 6m + 7$. [2]

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(b) Factorise $6x^2 - 12x$. [2]

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(a) Factorise $81p^2 - 1$. [2]

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(b) Factorise $7t^2 + 19t - 6$. [2]

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Rearrange the following formula to make x the subject. [3]

$$cx - 3 = 4x + d$$

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Make a the subject of the following formula.

[5]

$$\frac{2a^2 - b}{a^2b} = 1$$

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Make y the subject of the following formula.

[4]

$$2y = \sqrt{3 + my^2}$$

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A solution of the equation

$$x^3 - 3x = 37$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

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A solution to the equation

$$2x^3 - 3x - 17 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

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A solution of the equation

$$x^3 - 5x - 350 = 0$$

lies between 7.2 and 7.3.

Use the method of trial and improvement to find this solution correct to 2 decimal places.
You must show all your working.

[4]

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A solution of the equation

$$2x^3 + x - 10 = 0$$

lies between 1 and 2.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

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A solution to the equation

$$x^3 - 2x - 45 = 0$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

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Solve the equation $\frac{2x-3}{5} + \frac{4x+5}{2} = \frac{11}{2}$.

[4]

Solve the equation $\frac{4x-3}{2} + \frac{7x+1}{6} = \frac{29}{2}$.

[4]

. Express the following as a single fraction in its simplest form.

[3]

$$\frac{6}{3x-5} - \frac{4}{2x+1}$$

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Simplify the following expression.

[4]

$$\frac{2x^2 - 13x + 20}{2x - 8}$$

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- (a) Write the following expression as a single fraction.
Give your answer in its simplest form.

[2]

$$\frac{1}{x-a} - \frac{1}{x}$$

- (b) Solve the following equation.

[5]

$$\frac{x-1}{x(4x+3)} + 2 = 0$$

Express $\frac{3x}{3x+2} - \frac{2x}{2x+7}$ as a single fraction in its simplest form.

[3]

Given that y is inversely proportional to x , and that $y = 4$ when $x = 3$,

(a) find an expression for y in terms of x , [3]

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(b) use the expression you found in (a) to complete the following table. [2]

x	3	0.25	
y	4		$\frac{1}{5}$

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A farmer knows that the time, t , taken by goats to eat all the grass in a particular field is inversely proportional to the number of goats, g , in the field.

When there are 25 goats in the field, the time taken to eat all the grass is 36 days.

You may assume that all the goats eat grass at the same rate.

- (a) Find a formula for the time, t , in terms of the number of goats, g . [3]

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- (b) Hence, find the time taken for all of the grass to be eaten when there are 20 goats in the field. [1]

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- (c) The farmer needs the grass to last for at least 40 days.
What is the greatest number of goats that should be allowed in the field? [2]

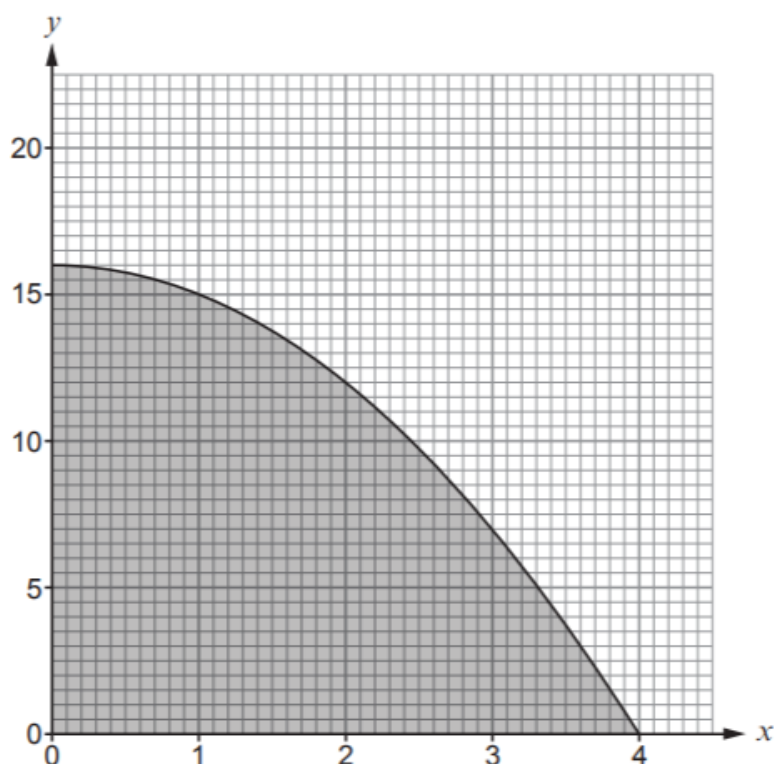
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The graph of $y = 16 - x^2$ has been drawn below, for values of x from $x = 0$ to $x = 4$.



- (a) Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$ and $x = 4$, to estimate the area of the shaded region shown above. [3]

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- (b) The area can be estimated again, using ordinates at every half unit, namely $x = 0$, $x = 0.5$, $x = 1$, $x = 1.5$, $x = 2$, $x = 2.5$, $x = 3$, $x = 3.5$ and $x = 4$.

Without calculating the new area, tick one of the following boxes.

The new area will be **equal to** the estimated area found in part (a).

☐

The new area will be **greater than** the estimated area found in part (a).

☐

The new area will be **less than** the estimated area found in part (a).

☐

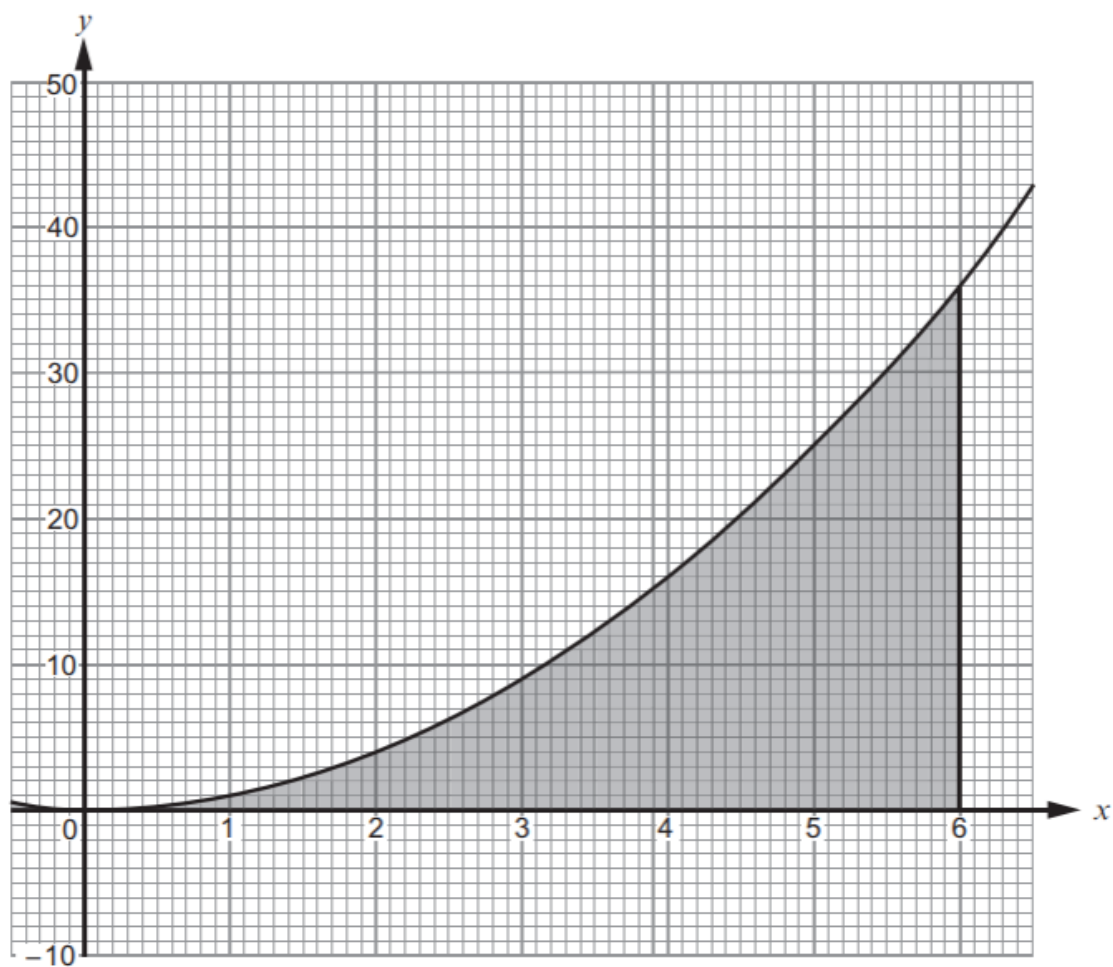
You must give a reason for your answer.

[1]

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The graph of $y = x^2$ has been drawn below, for values of x from $x = 0$ to $x = 6$.



Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$, $x = 5$ and $x = 6$, to estimate the area of the shaded region shown above. [4]

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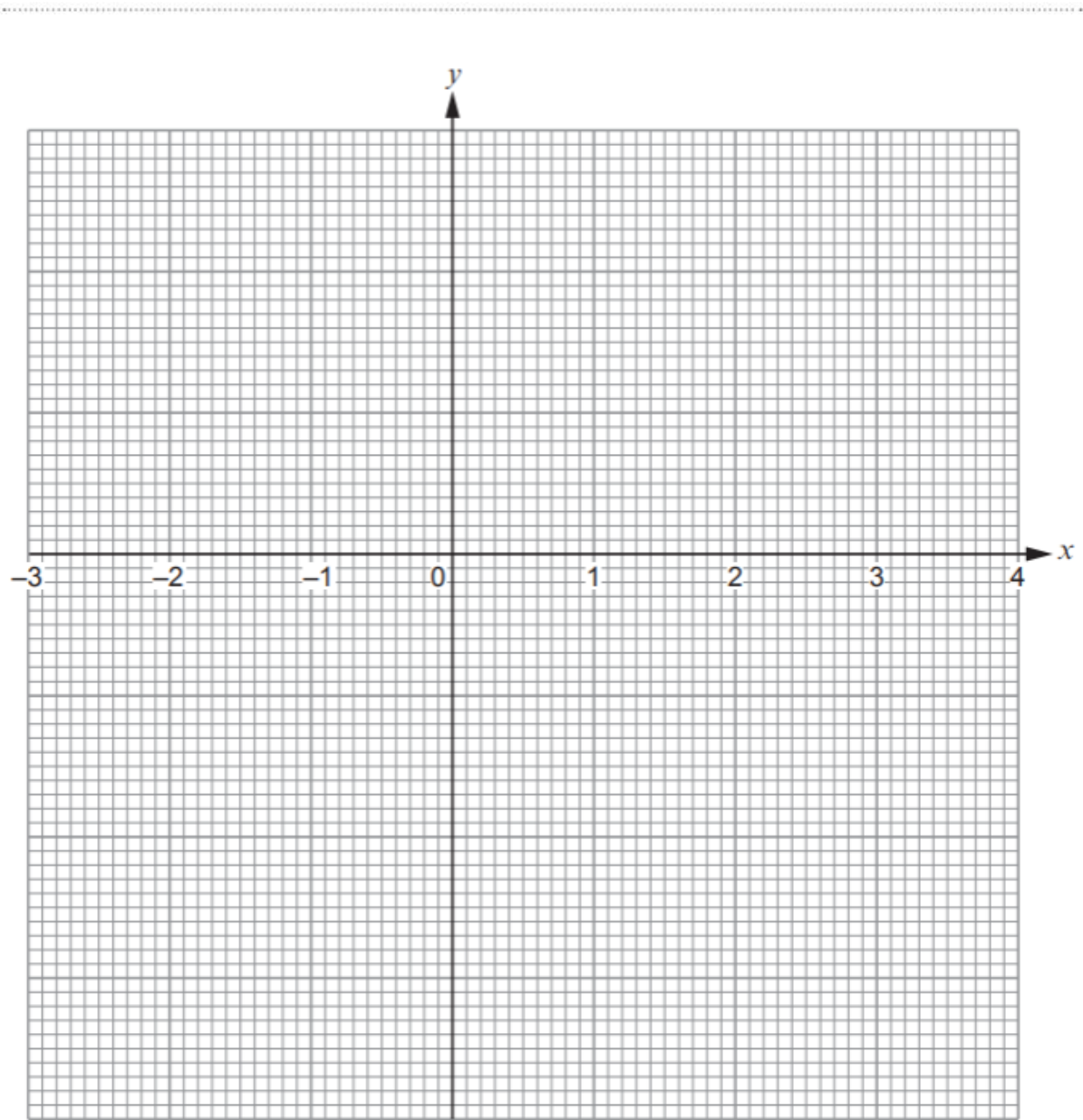
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Complete the table below.
Draw the graph of $y = 3x^2 - 25$ for values of x between -3 and 4 .
Use the graph paper below.
You must choose a suitable scale for the y -axis.

[4]

x	-3	-2	-1	0	1	2	3	4
$y = 3x^2 - 25$	2		-22	-25	-22	-13	2	23



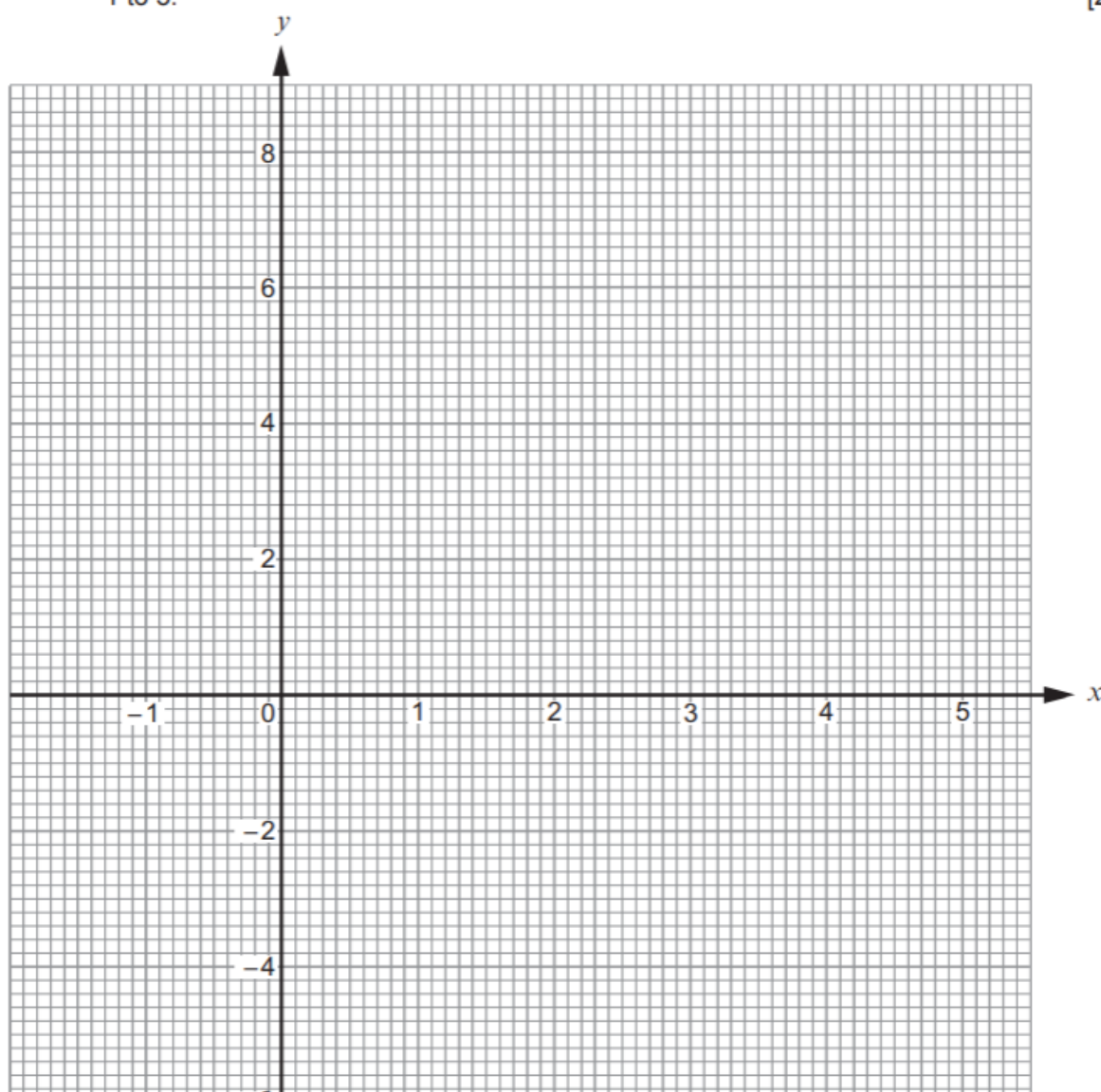
The table below shows some of the values of $y = x^2 - 5x + 2$, for values of x from -1 to 5 .

x	-1	0	1	2	3	4	5
$y = x^2 - 5x + 2$	8	2	-2	-4		-2	2

- (a) Complete the table above. [1]

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- (b) On the graph paper below, draw the graph of $y = x^2 - 5x + 2$ for values of x from -1 to 5 . [2]



- (c) Draw the line $y = -3$ on the graph paper.

Write down the values of x where the line $y = -3$ cuts the curve $y = x^2 - 5x + 2$.
 Give your answers correct to 1 decimal place.

[2]

Values of x are and

- (a) The table below shows some of the values of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

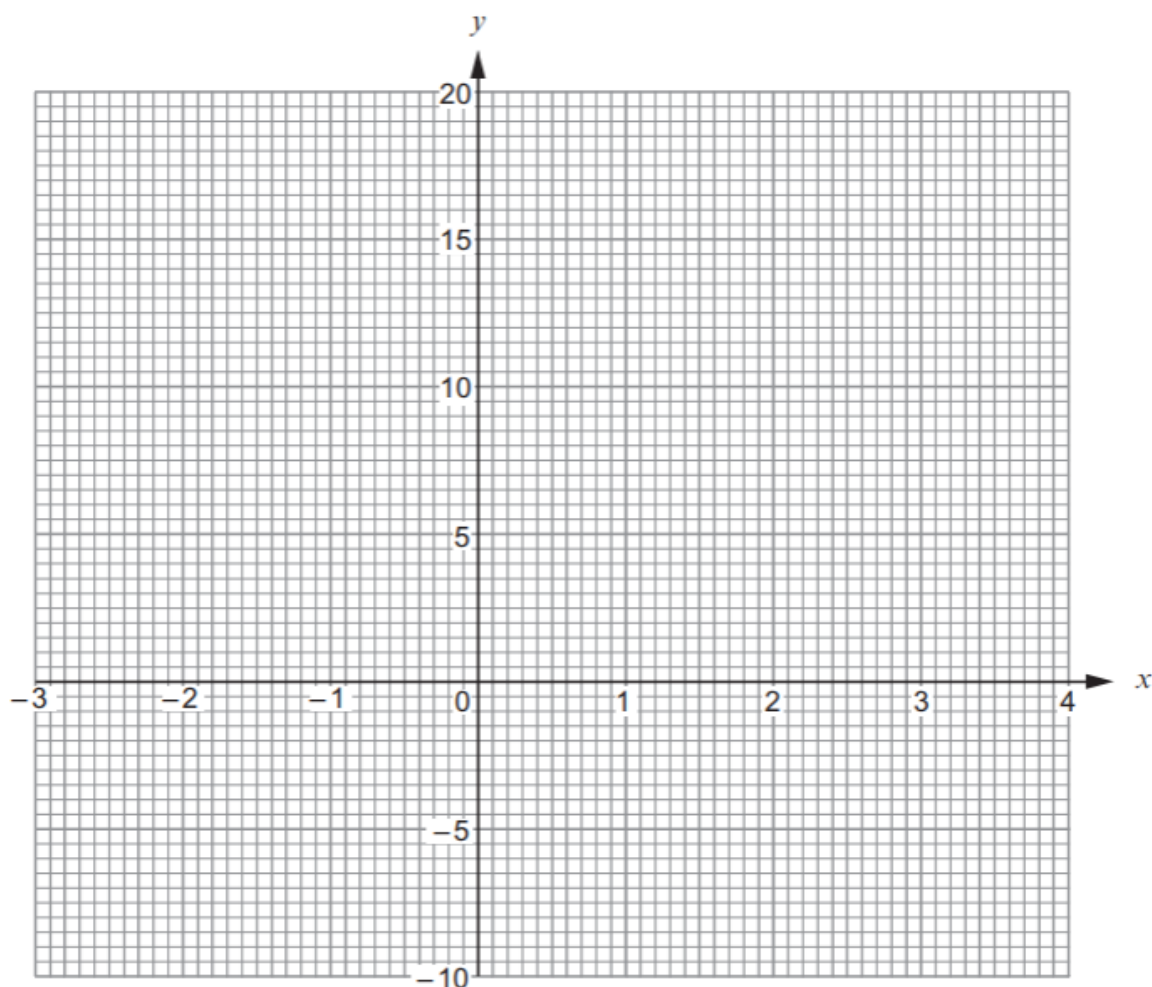
Complete the table by finding the value of y for $x = -1$ and for $x = 2$.

[2]

x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x - 1$	17		-1	-4		2	11

- (b) On the graph paper below, draw the graph of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

[2]



- (c) Draw the line $y = 5$ on the graph paper.

Write down the values of x where the line $y = 5$ cuts the curve $y = 2x^2 - 5x - 1$.
Give your answers correct to 1 decimal place.

[2]

Values of x are and

- (d) Circle the equation below whose solutions are the values you have given in (c).

[1]

$$2x^2 - 5x - 1 = 0$$

$$2x^2 - 5x - 6 = 0$$

$$2x^2 - 5x - 5 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 5x + 4 = 0$$

3. The table below shows some of the values of $y = x^2 - 4x - 3$ for values of x from -2 to 5 .

x	-2	-1	0	1	2	3	4	5
$y = x^2 - 4x - 3$		2	-3	-6		-6	-3	2

- (a) Complete the table by finding the value of y for $x = -2$ and the value of y for $x = 2$. [2]

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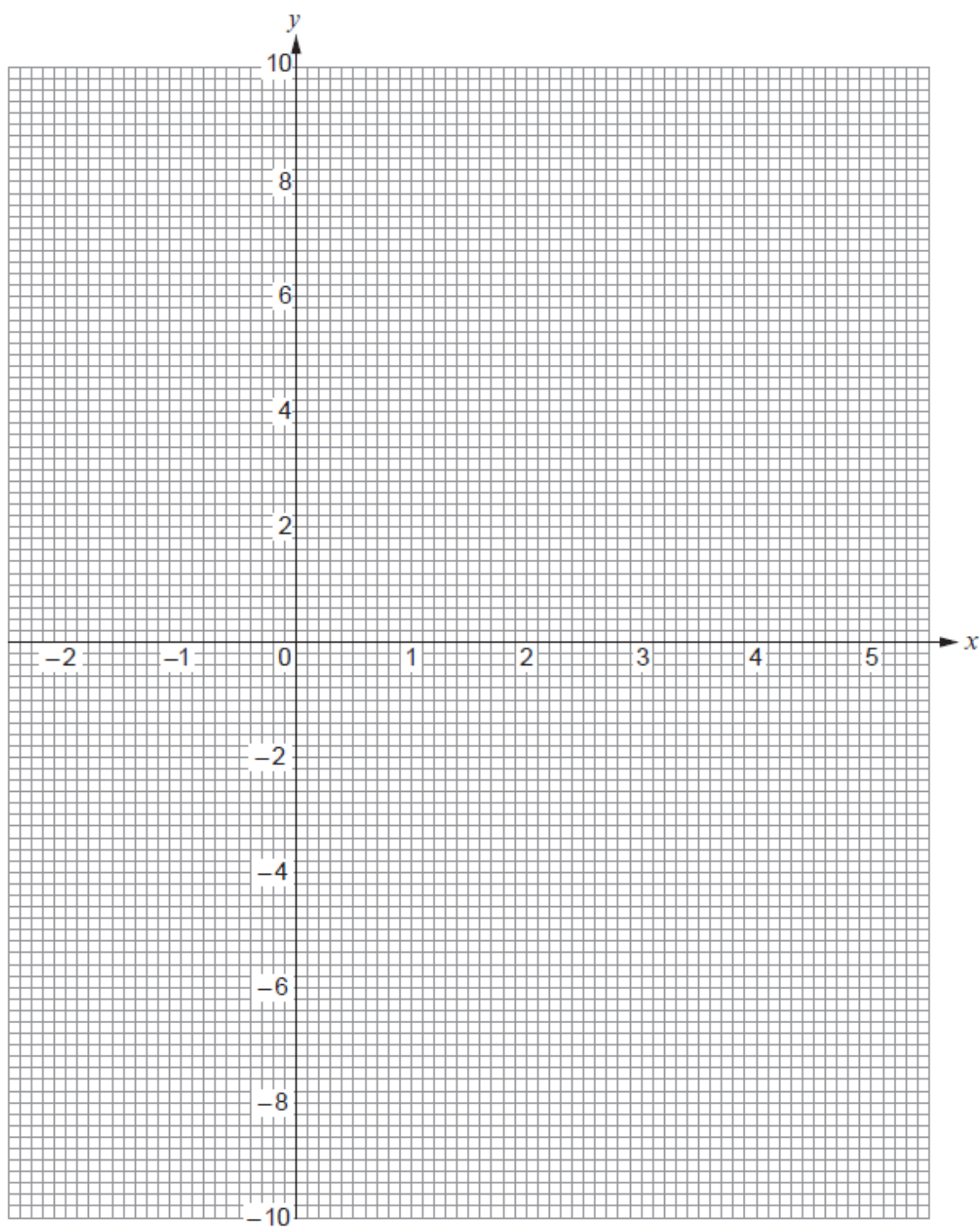
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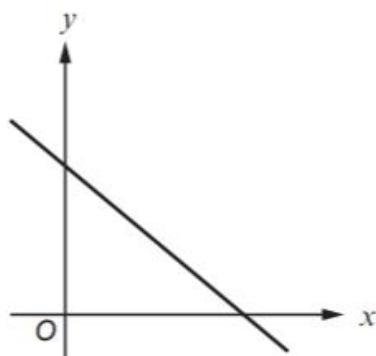
- (b) On the graph paper opposite, draw the graph of $y = x^2 - 4x - 3$ for values of x from -2 to 5 . [2]

- (c) Draw the line $y = 1$ on the graph paper.
Write down the values of x where the line $y = 1$ cuts the curve $y = x^2 - 4x - 3$. [2]

Values of x are and



(a)



Which **one** of the following equations could represent the line shown in the graph above?
Circle your answer. [1]

$y = -x - 2$ $y = -x + 2$ $y = x + 2$ $y = x - 2$ $y = -x$.

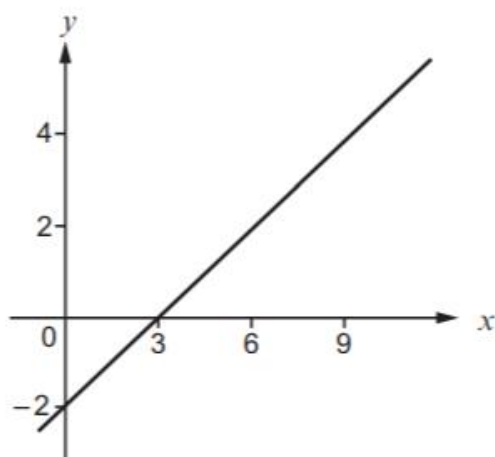
(b) Which **one** of the following points lies on the line $2y = 3x + 4$?
Circle your answer. [1]

$(2, -5)$ $(5, 2)$ $(-2, 5)$ $(2, 5)$ $(-2, -5)$

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(c)



What is the gradient of the line shown in the graph above?
Circle your answer. [1]

$\frac{3}{2}$ $-\frac{3}{2}$ $\frac{2}{3}$ $-\frac{2}{3}$ -6

The table below shows some of the values of $y = x^2 - 2x - 4$ for values of x from -3 to 4 .

x	-3	-2	-1	0	1	2	3	4
$y = x^2 - 2x - 4$	11	4	-1	-4		-4	-1	4

- (a) Complete the table by finding the value of y when $x = 1$. [1]

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- (b) On the graph paper opposite, draw the graph of $y = x^2 - 2x - 4$ for values of x from -3 to 4 . [2]

- (c) (i) Draw the line $y + x = 4$ on the graph paper. [2]

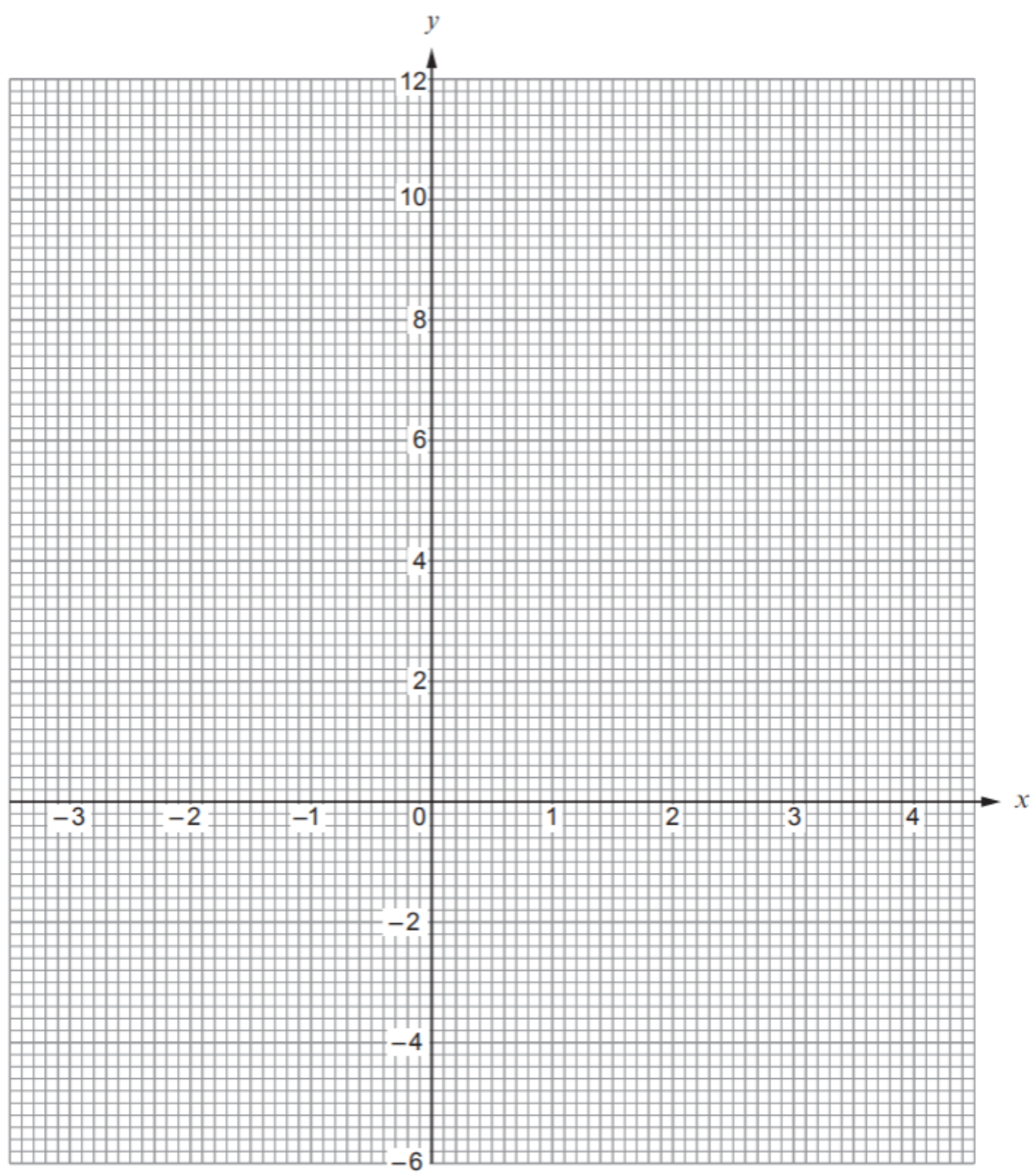
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- (ii) Write down the values of x where the line $y + x = 4$ cuts the curve $y = x^2 - 2x - 4$. [1]

Values of x are and



The equation of a straight line is $y = 8x - 5$.
What is the gradient of the line?

Circle the correct answer.

[1]

$\frac{1}{8}$

-5

8

5

1

9. (a) Circle the equation of a straight line that is parallel to the line $3y = 2x + 6$.

[1]

$3y = 2x + 7$

$2y = 3x + 6$

$3y = -2x + 6$

$-3y = 2x + 6$

$2y = -3x + 6$

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- (b) Circle the equation of a straight line that is perpendicular to the line $y = 5x - 3$.

[1]

$y = \frac{x}{5} + 3$

$y = 5x + 3$

$y = 5x + \frac{1}{3}$

$y = -5x + 3$

$y = \frac{-x}{5} + 3$

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- (a) Which one of the following equations represents a straight line that is parallel to the line $2y = 5x - 4$?

Circle your answer.

[1]

$y = 2.5x + 3$

$y = 5x - 2$

$y = 0.4x - 4$

$y = -0.4x - 2$

$2y = -5x + 4$

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- (b) Which one of the following equations represents a straight line that intersects the line $y = 7x - 5$ on the y -axis?

Circle your answer.

[1]

$y = 7x + 5$

$y = 5 - 7x$

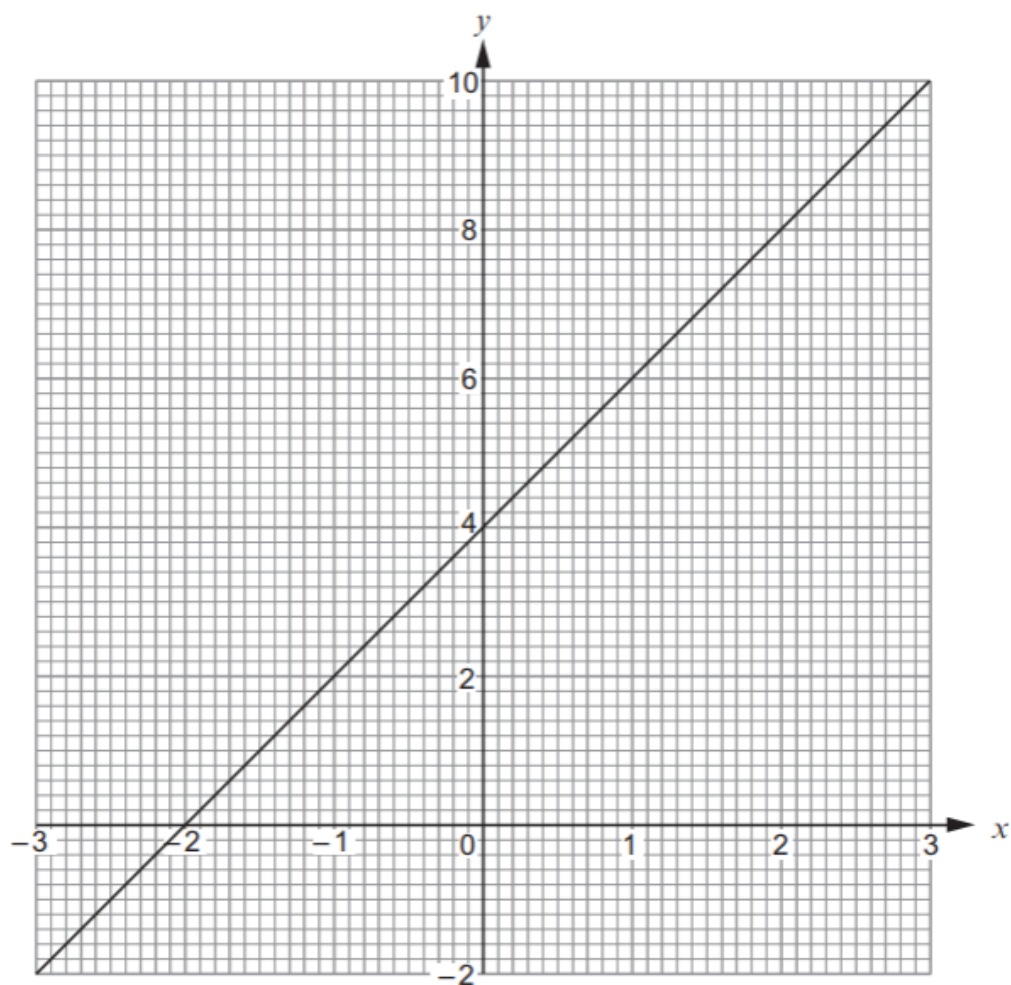
$y = 3x + 5$

$y = 0$

$y = 3x - 5$

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- (a) The diagram below shows the graph of a straight line for values of x from -3 to 3 .

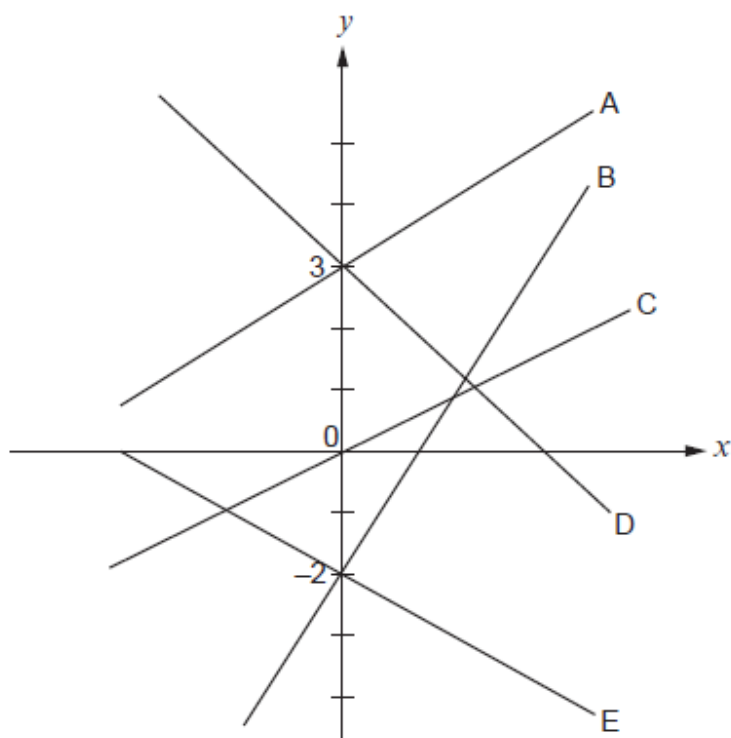


- (i) Write down the gradient of the above line. [1]

- (ii) Write down the equation of the line in the form $y = mx + c$, where m and c are whole numbers. [2]

- (b) Without drawing, show that the line $2y = 5x - 3$ is parallel to the line $4y = 10x + 7$. You must show working to support your answer. [2]

(c)



Which one of the five straight lines shown above could represent the equation $y = -2x + 3$?
Circle your answer.

[1]

Line A

Line B

Line C

Line D

Line E

William has n marbles.

Lois had 4 times as many marbles as William, but she has now lost 23 of them.

Lois still has more marbles than William.

Write down an inequality in terms of n to show the above information.

Use your inequality to find the least number of marbles that William may have.

[4]

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Arthur, Sian and Kezia are all given some £1 coins.

Arthur receives £ n .

Sian is given five times as much money as Arthur.

Kezia receives three times as much money as Arthur, plus an extra £7.

Sian was given less money than Kezia.

- (a) Write down an inequality in terms of n that illustrates the fact that Sian received less money than Kezia. [2]

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- (b) What was the greatest amount of money that Arthur could have been given? [2]

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In this question, you will be assessed on the quality of your linguistic and mathematical accuracy in writing.

Rashid owned n sheep.

Eifion had exactly 4 times as many sheep as Rashid.

Rashid buys 17 extra sheep.

Eifion sells 8 of his sheep.

Eifion still has more sheep than Rashid.

Form an inequality, in terms of n .

Solve the inequality to find the **least** value of n .

You must show all your working.

[5 + 1 W]

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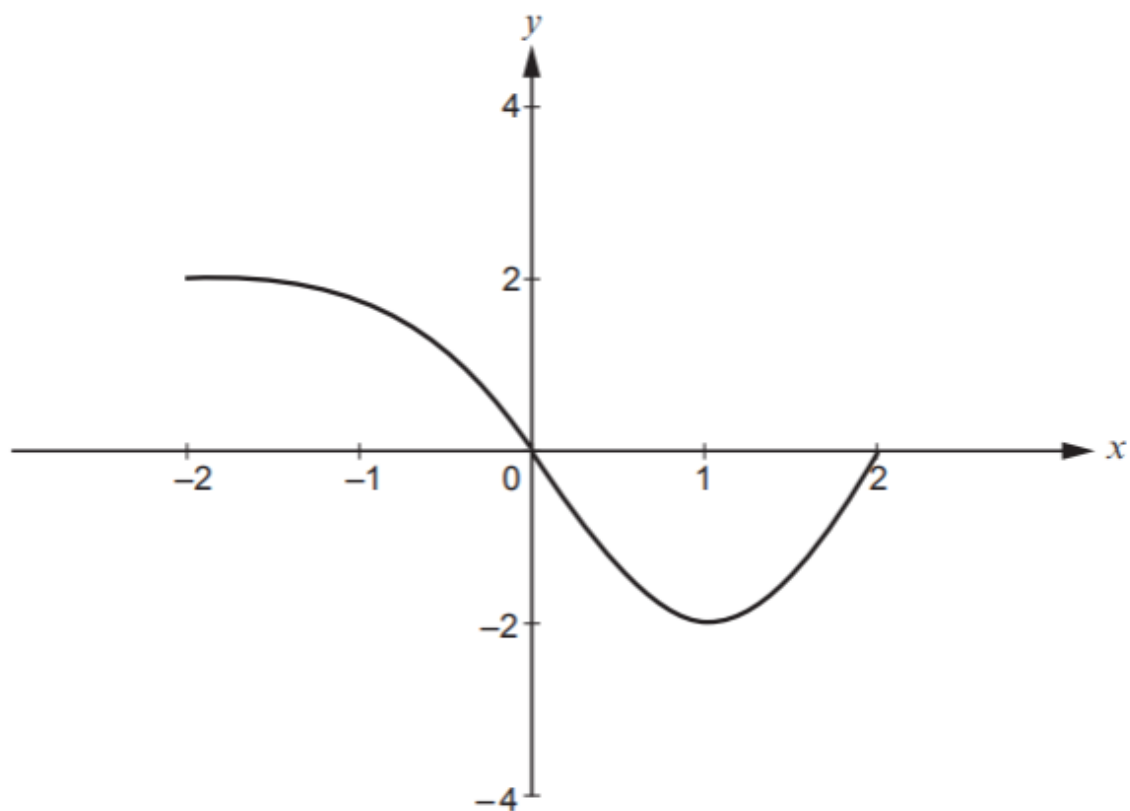
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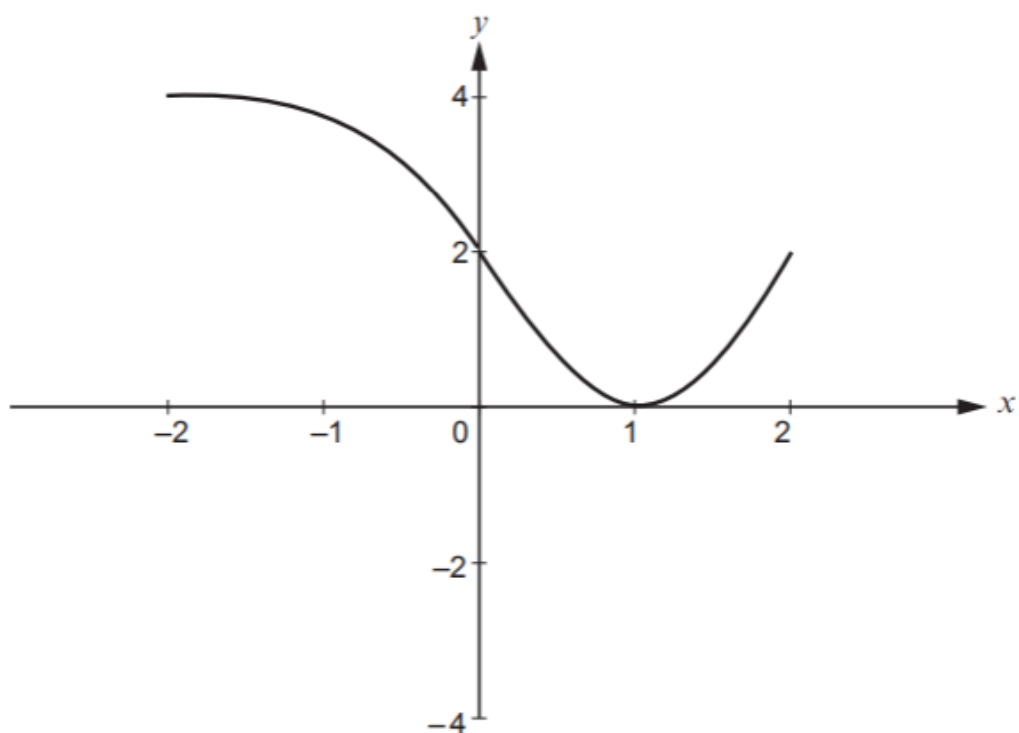
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The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.

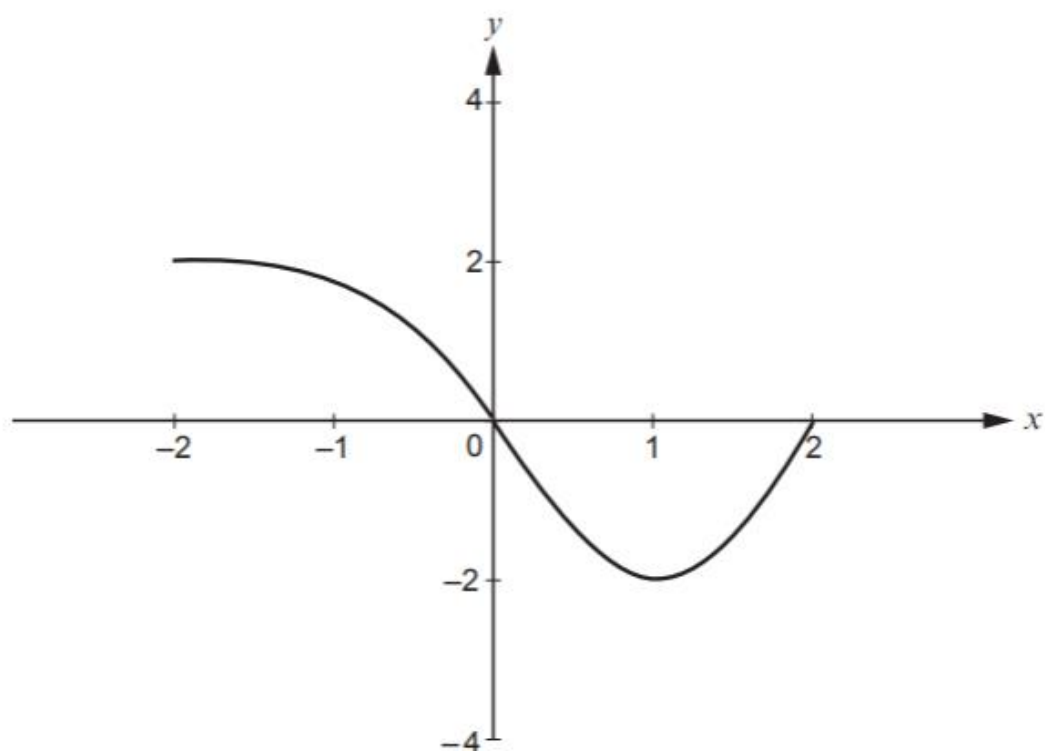


Using function notation, complete the equation of the transformed curve.

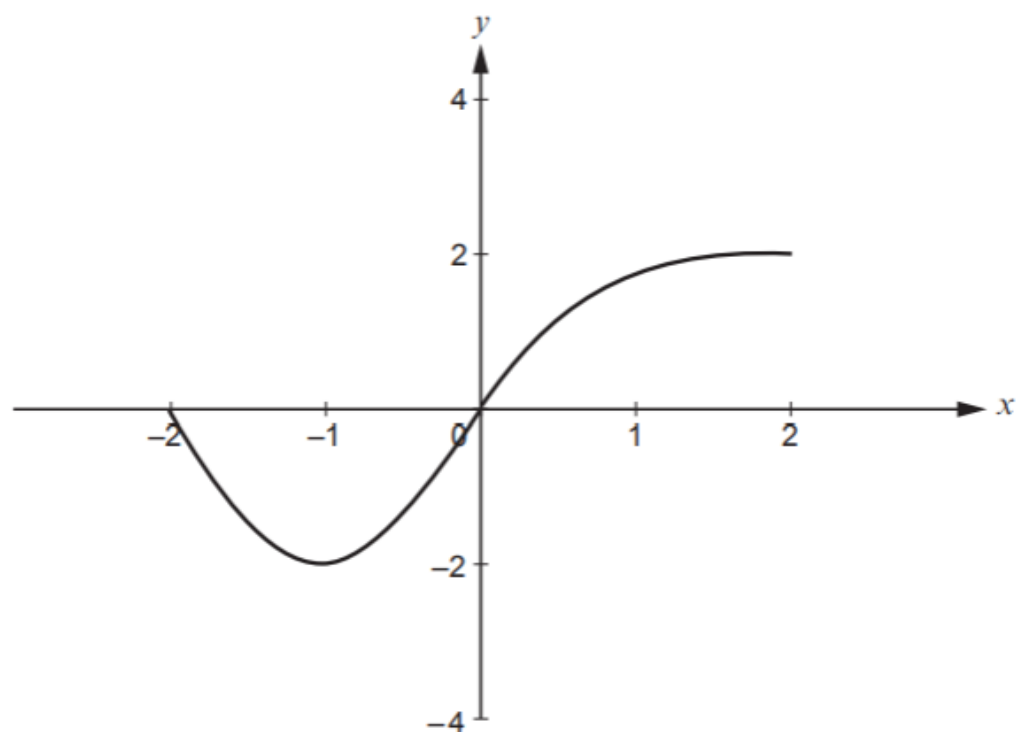
[1]

$y = \dots\dots\dots$

(b) The following diagram again shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



Using function notation, complete the equation of the transformed curve.

[1]

$$y = \dots\dots\dots$$

- (a) Using the axes below, **sketch** the graph of $y = \sin x$ for values of x from 0° to 360° . You must label any important values on both axes. [2]



- (b) Circle the value that is equal to $\sin 200^\circ$. [1]

$\sin 20^\circ$

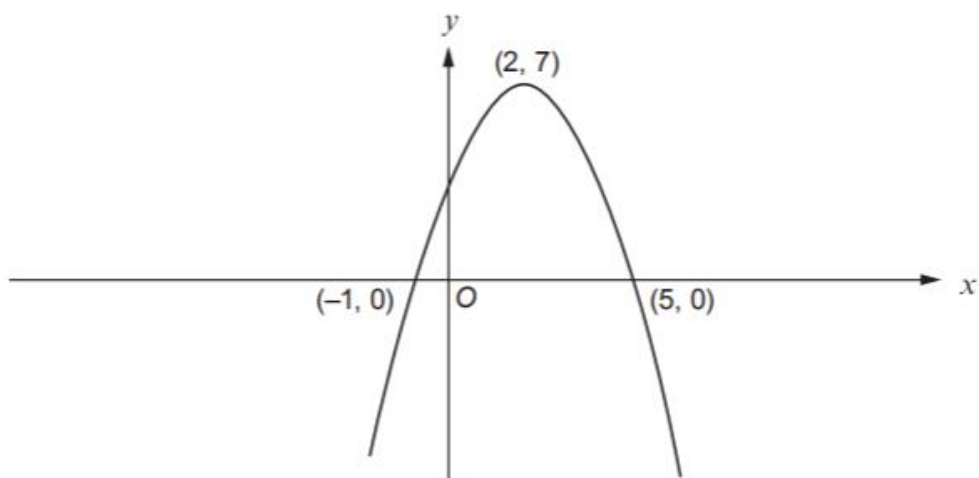
$\sin 100^\circ$

$\sin 160^\circ$

$\sin 220^\circ$

$\sin 340^\circ$

- (a) The diagram shows a sketch of the graph $y = f(x)$.
The graph passes through the points $(-1, 0)$ and $(5, 0)$ and its highest point is at $(2, 7)$.

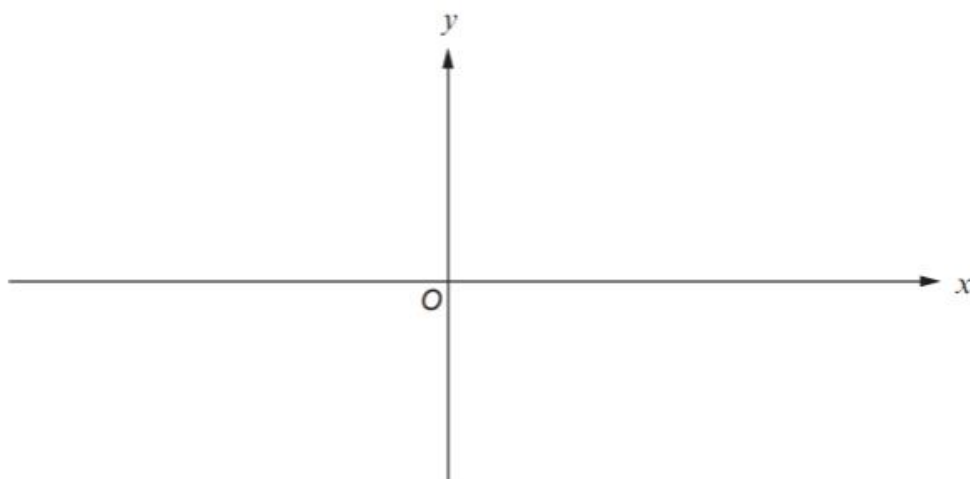


Sketch the graph of $y = f(x - 3)$ on the axes below.

You must indicate

- the coordinates of the points of intersection of the graph with the x -axis
- the coordinates of the highest or lowest point.

[3]

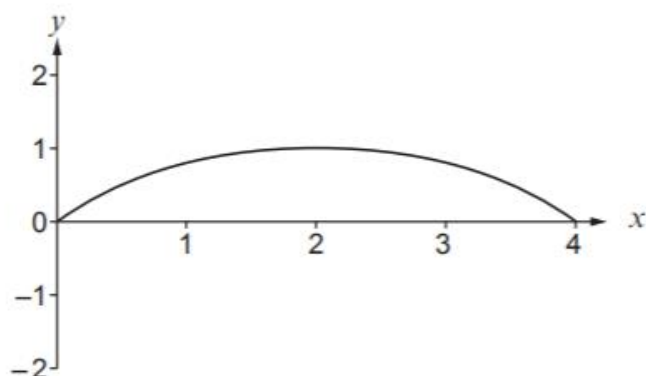


- (b) Using the axes below, **sketch** the graph of $y = \cos x + 1$ for values of x from 0° to 360° .

[2]



The following graph represents the curve $y = f(x)$.

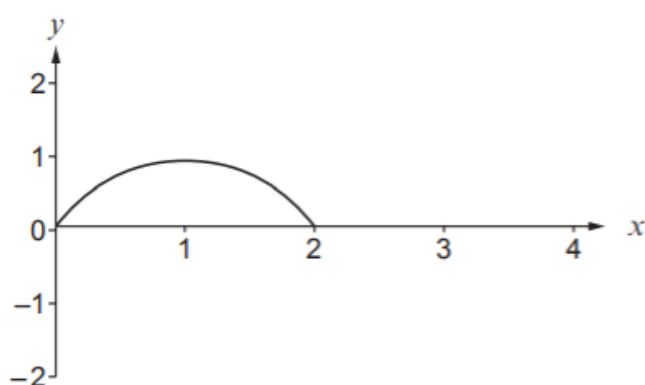


There are 9 equations and 4 graphs of different transformations of $y = f(x)$ shown below. Choose the equation which describes each transformation of $y = f(x)$. Write it in the box provided.

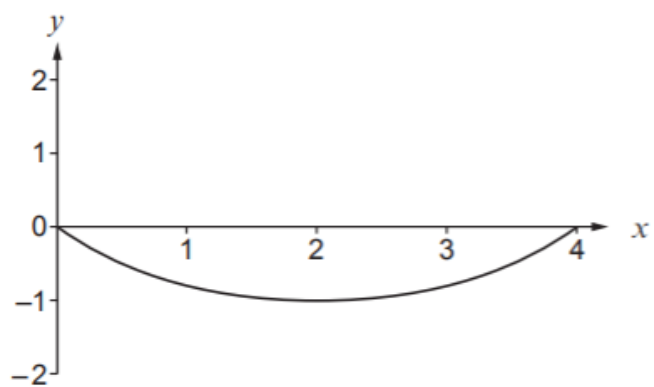
One has been completed for you.

[3]

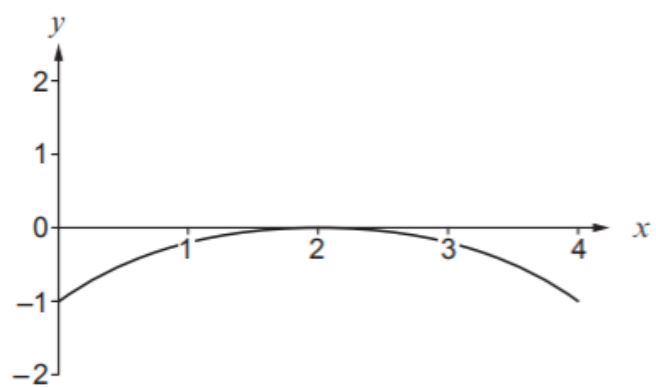
<u>Equations</u>		
$y = f(2x)$	$y = f(x - 1)$	$y = f(x) - 1$
$y = -f(x)$	$y = f(x) + 1$	$y = f(-x)$
$y = \frac{1}{2}f(x)$	$y = 2f(x)$	$y = -2f(x)$



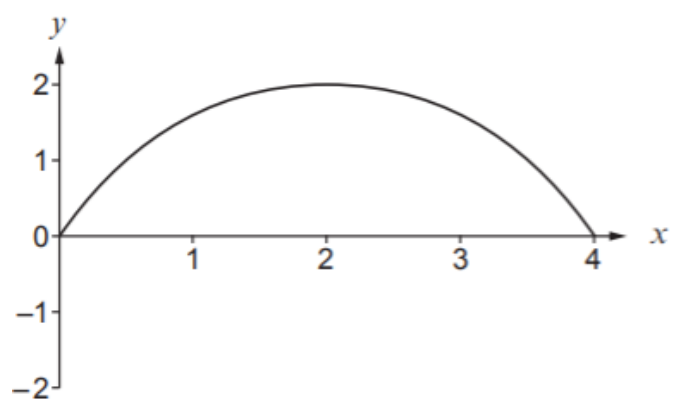
Equation: $y = f(2x)$



Equation:



Equation:



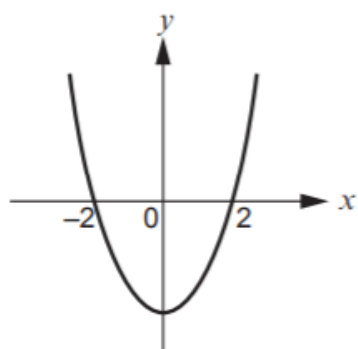
Equation:

Four quadratic graphs are sketched below.
 Draw a line connecting each graph to its equation.
 One has been completed for you.

[2]

Graph

Equation



$$y = (x + 1)(x - 4)$$

$$y = (x - 4)^2$$

$$y = x(x + 4)$$

$$y = (x - 1)(x + 4)$$

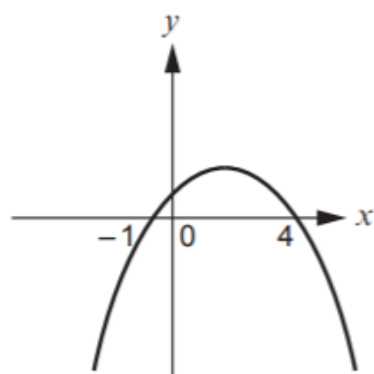
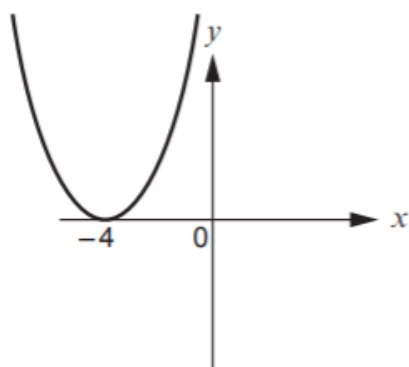
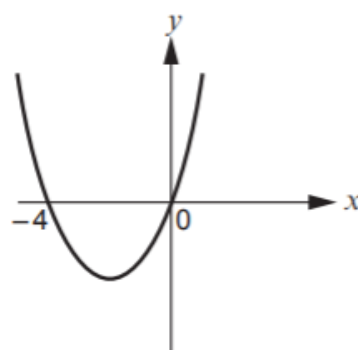
$$y = (x - 2)(x + 2)$$

$$y = x(x - 4)$$

$$y = (x + 1)(4 - x)$$

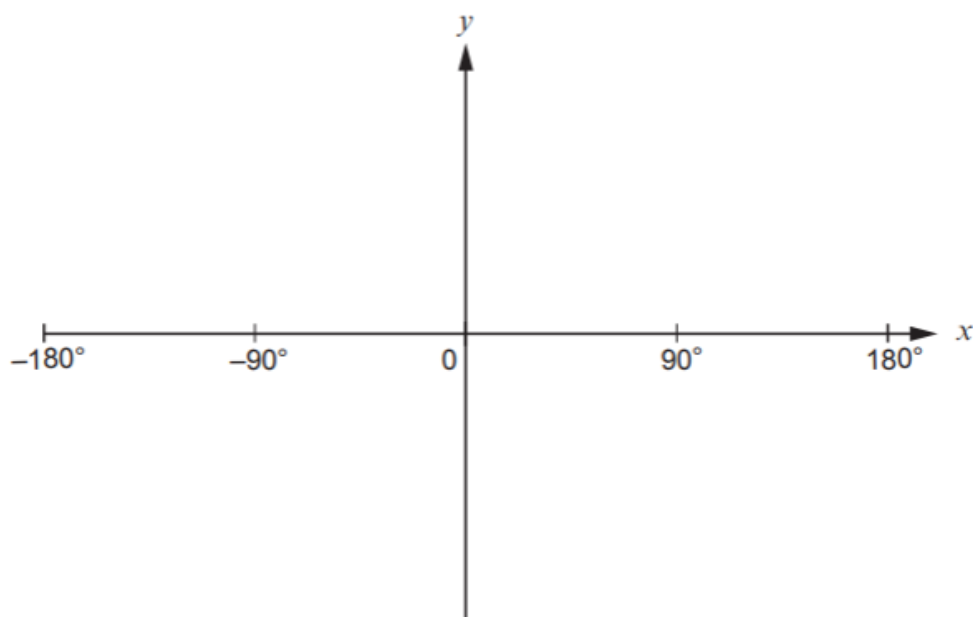
$$y = (1 - x)(x + 4)$$

$$y = (x + 4)^2$$



- (a) Sketch the curve $y = \sin x$ on the axes below.
You must indicate any important values on the y -axis.

[2]

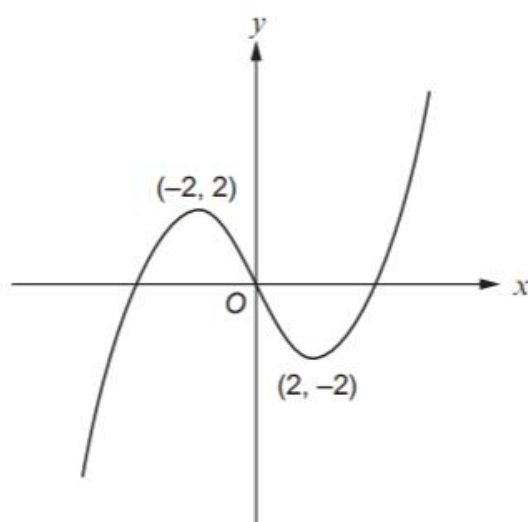


- (b) Solve the equation $\sin x = -0.5$.
Give all answers in the range $x = -180^\circ$ to $x = 180^\circ$.

[2]

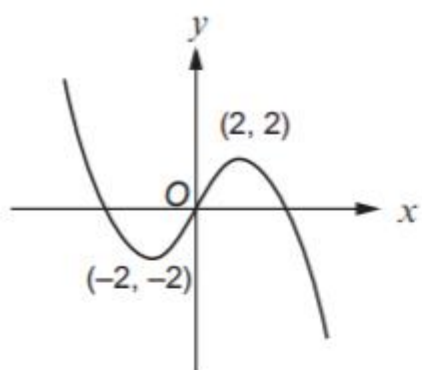
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- A sketch of the graph $y = f(x)$ is shown below.
Two specific points are shown on the graph. They are called a maximum point and a minimum point.
The maximum point shown is $(-2, 2)$ and the minimum point shown is $(2, -2)$.



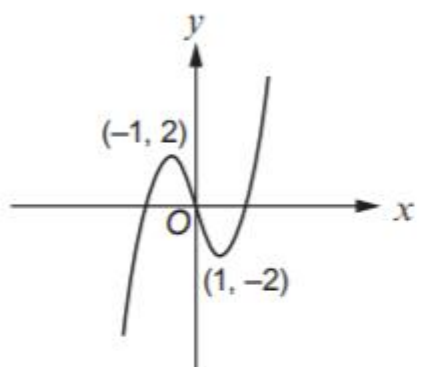
The graphs on the opposite page are transformations of $y = f(x)$.
Draw a line connecting each graph to the equation describing the transformation.
One has been done for you.

[4]

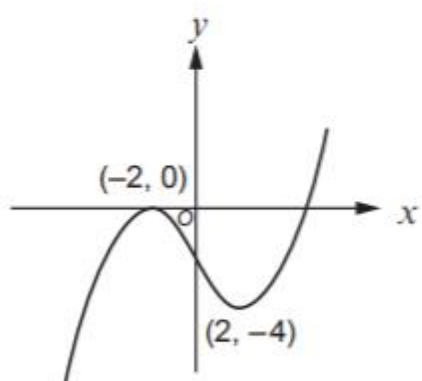


$$y = f(x) - 2$$

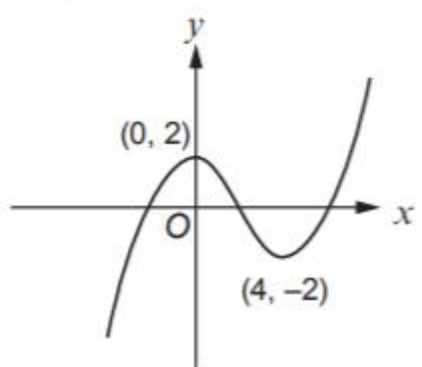
$$y = f(x + 2)$$



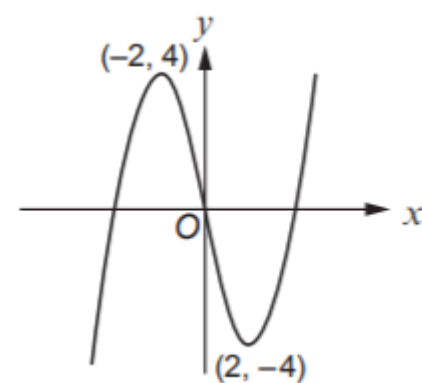
$$y = -f(x)$$



$$y = 2f(x)$$



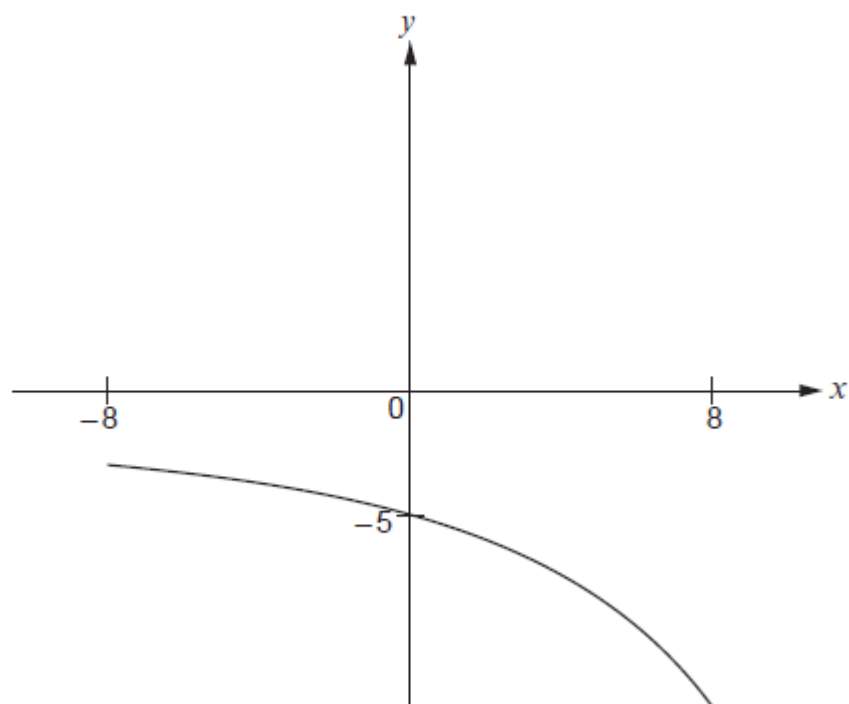
$$y = f(x) + 2$$



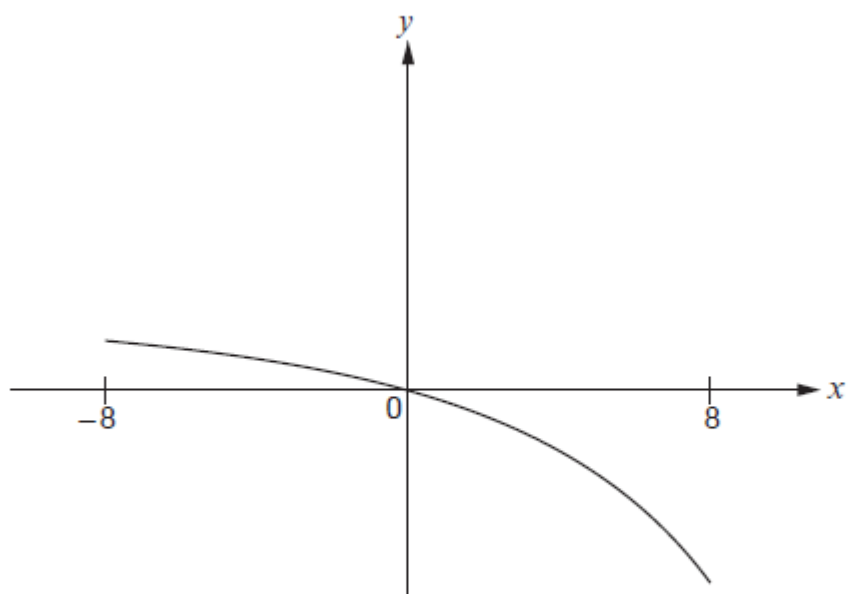
$$y = \frac{1}{2} f(x)$$

$$y = f(x - 2)$$

The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.

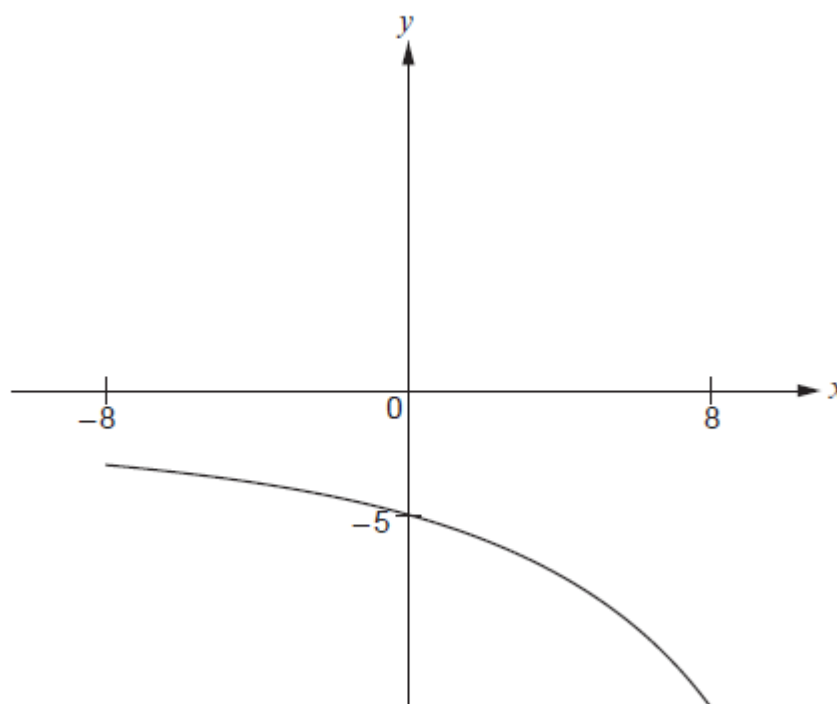


Using function notation, complete the equation of the transformed curve.

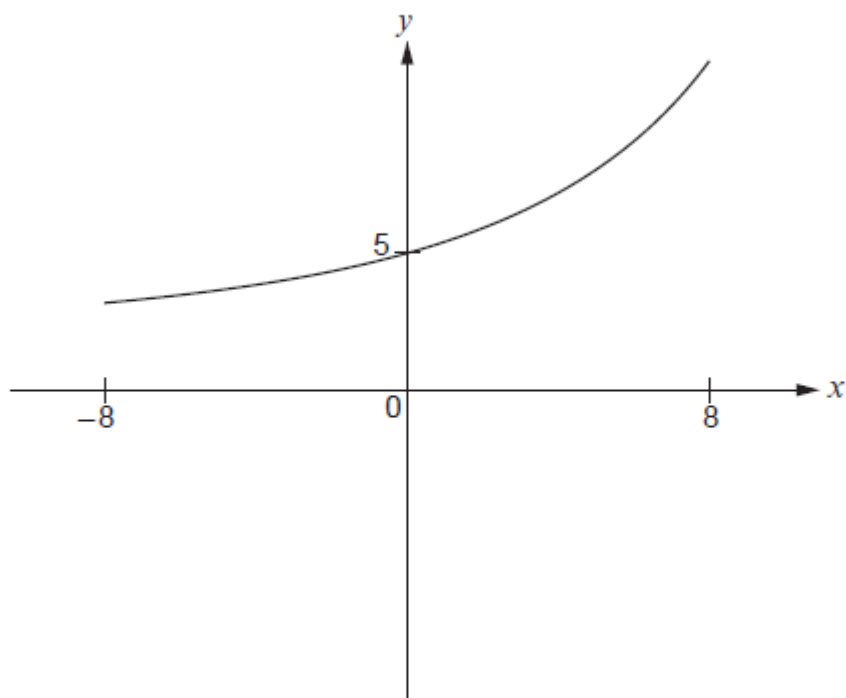
[1]

$$y = \dots\dots\dots$$

The following diagram again shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



Using function notation, complete the equation of the transformed curve.

[1]

$$y = \dots\dots\dots$$

Sketch the curve $y = \cos x$, for values of x in the range $x = 0^\circ$ to $x = 360^\circ$.
You must indicate any important values on the axes.

[2]



Solve the equation $\cos x = 0.7$.
Give all solutions in the range $x = 0^\circ$ to $x = 360^\circ$.

[2]

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[3]

[6]

The three concrete slabs cover an area of 7 m^2 .

[1]

[4]

[5]

The diagram shows two rectangles.

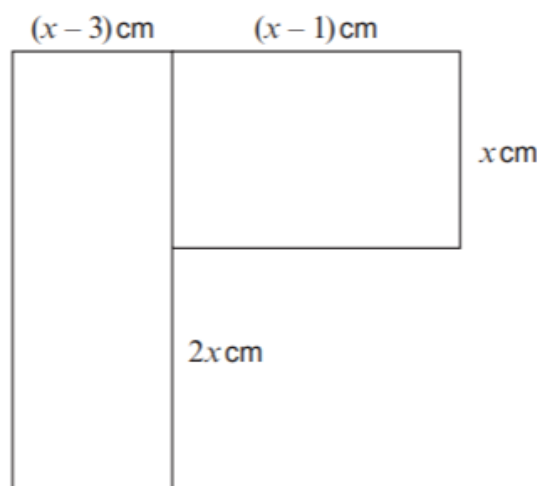


Diagram not drawn to scale

The combined area of both rectangles is 50 cm^2 .

By considering the areas of the two rectangles, show that $2x^2 - 5x - 25 = 0$ and hence find the value of x . [6]

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By considering algebraic expressions, show that it will never be possible for the surface area of a sphere of radius r to be equal to the surface area of a cube with sides of length r . [2]

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The area of the trapezium $ABCD$ is 25 cm^2 .

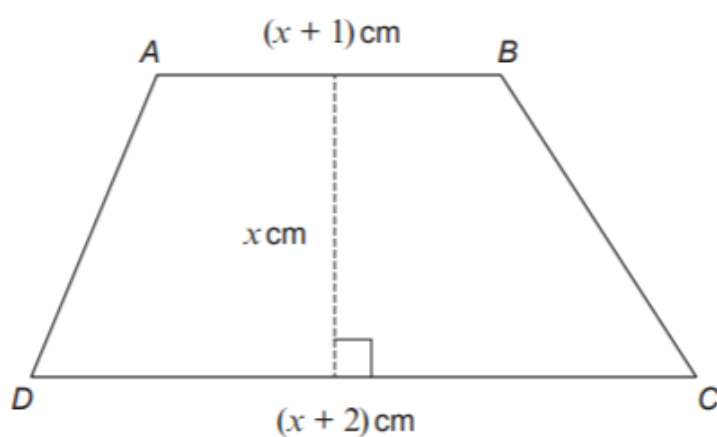


Diagram not drawn to scale

- (i) Show that $2x^2 + 3x - 50 = 0$. [3]

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- (ii) Solve the equation $2x^2 + 3x - 50 = 0$ to calculate the lengths AB and DC . Give your answers correct to 1 decimal place. [4]

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Show that the triangle below is **not** a right-angled triangle.

[5]

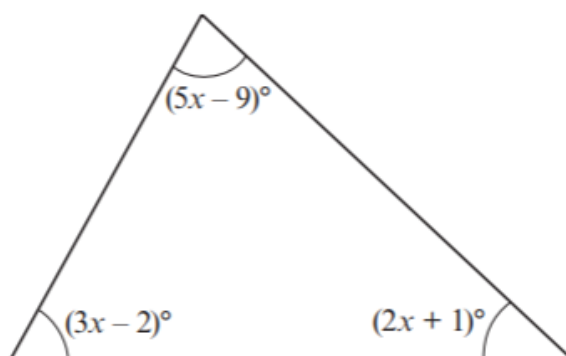


Diagram not drawn to scale

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